International Journal of Theoretical Physics, Vol. 20, No. 5, 1981

Nonlocal Stochastic Model for the Free Scalar Field Theory

Kh. Namsrai

Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna, 141980 USSR

Received November 3, 1980

The free scalar field is investigated within the framework of the Davidson stochastic model and of the hypothesis on space-time stochasticity. It is shown that the resulting Markov field obtained by averaging in this space-time is equivalent to a nonlocal Euclidean Markov field with the times scaled by a common factor which depends on the diffusion parameter ν. Our result generalizes Guerra and Ruggiero's procedure of stochastic quantization of scalar fields. On the basis of the assumption about unobservability of ν in quantum field theory, the Efimov nonlocal theory is obtained from Euclidean Markov field with form factors of the class of entire analytical functions.

1. INTRODUCTION

Interest in stochastic processes and fields has grown in the last years. This is mainly due to the fact that the close correspondence of stochastic processes to quantum mechanics (Nelson, 1966, 1967; Kershaw, 1964; de la Pena-Auerbach and Cetto, 1975; see also Davidson, 1979a, b; Lee, 1980) and also to Euclidean quantum field theory (Guerra and Ruggiero, 1973; Nelson, 1973) has been found. Generalizing the idea of stochastic quantization of Nelson (1966, 1967) and Fenyes (1952) to the case of continuous systems, Guerra and Ruggiero (1973) (see also Dankel, 1970) constructed the Euclidean field theory. These ideas have been developed further by Davidson (1980). The Euclidean field theory was finally formulated in the language of stochastic processes by Nelson (1973). At present the problem of simple correspondence between Euclidean and pseudo-Euclidean Green's functions (Osterwalder and Schrader, 1973, 1975; Glaser, 1974) investigations which were started by Schwinger (1959) and Nakano (1959) is solved too.

0020-7748/81/0500-0365$03.00/0

There are also other approaches to investigation of stochastic processes and fields (for example, see reviews of Moore, 1979; Boyer, 1975; Surdin, 1971, 1978; Blokhintsev, 1975). Some of them start with the hypothesis on the stochastic properties of the electromagnetic vacuum (Braffort and Tzara, 1954; Marshall, 1963) and of the space-time (March, 1934, 1937; Frederick, 1976). Earlier papers (Namsrai, 1980a, 1980c) in which dynamics of particles and relativistic Feynman-type integrals have been investigated concern the latter approach.

In this paper we shall study the free scalar field within the framework of the Davidson (1980) stochastic model and of the hypothesis about the space-time stochasticity. The basic hypothesis (Namsrai, 1980a) is the following:

(i) The physical quantities are considered as functions of complex times $t+i\tau$ in the limit $\tau\to 0$.

(ii) The stochasticity of the space $R_4(\hat{x})$ appears in the Euclidean space $(\mathbf{x},\tau)$ but not in the Minkowski space $(\mathbf{x},t)$. So in our model the actual points of the space $R_4(\hat{x})$ consist of two parts:

$$\hat{x}=(\mathbf{x}+\mathbf{b}, x_0+i\tau), \qquad x_0=ct$$

where $x=(\mathbf{x},ct)$ is the regular part and $b_E=(\mathbf{b}, b_4=\tau)$ some small random vector with a distribution $\lambda(b_E^2/l^2)$ obeying the conditions

$$\int d\lambda(b_E^2/l^2)=1, \qquad d\lambda(b_E^2/l^2)\geqslant 0$$

Here $b_E^2=\mathbf{b}^2+b_4^2$ and the constant l has the dimension of length which we call fundamental length. From the physical point of view, the fundamental (or universal) length l characterizes a certain domain within which the existing space concepts and causality conditions may be violated but the stochastic properties or fluctuations in the metric can be manifested if they exist. The estimates given in the papers of Dineikhan and Namsrai (1977) and Kadyshevsky (1980) show that $l\lesssim 10^{-15}-10^{-16}$ cm. Some possibility of introduction of the concept of the fundamental length in physics is discussed by Kadyshevsky (1980), Ginzburg (1975), Hsu and Mac (1979), Fubini (1974), and Cheon (1978).

Since the points of the space $R_4(\hat{x})$ are of stochastic nature, neither of these points can be used as a basis for a coordinate system, nor can one take a derivative with respect to them. However, the space of common experience (i.e., the laboratory frame) is nonstochastic on a large scale.

Therefore a mathematical construction is needed for transition from the microworld to this large-scale nonstochastic space (see Frederick, 1976).

In our case this mathematical construction reduces to averaging with the distribution $\lambda(b_E^2/l^2)$ at any point of the space $R_4(\hat{x})$ at a given time. So, the averaged quantity $\langle f(\hat{x})\rangle$ on $R_4(\hat{x})$ with $\lambda(b_E^2/l^2)$ is called the physical value of $f(\mathbf{x}, t)$ (see Namsrai, 1980b, for detail).

2. THE FREE SCALAR FIELD IN STOCHASTIC SPACE

Now we pass to the study of the free scalar field in the space $R_4(\hat{x})$ within the framework of the Davidson (1980) model. Following Davidson we consider first the classical equation for the real free scalar field in the Minkowski space:

$$P_\mu P^\mu \phi = m^2 \phi, \qquad P_\mu = -i\hbar\partial/\partial x^\mu$$

where c has been set to unity and $g^{00}=1$. Let us require periodic boundary conditions on the field:

$$\phi(\mathbf{x}, t) = \phi(\mathbf{x}+\mathbf{a}, t), \qquad a_i = n_i L \qquad (n_i \text{ is an integer})$$

so that one may write:

$$\phi(\mathbf{x}, t) = (\hbar L^3)^{-1/2} \sum_{\mathbf{q}} e^{i\mathbf{q}\mathbf{x}} \phi_{\mathbf{q}}(t), \qquad k_i = 2\pi m_i/L \tag{1}$$

(m_i is an integer).

Further, as usual, it is assumed that each component of the Fourier decomposition (1) is a random variable satisfying the stochastic differential equation:

$$d\phi_{\mathbf{q}}(t) = b_{\mathbf{q}}(\phi_{\mathbf{q}}(t))\, dt + dW_{\mathbf{q}}(t) \tag{2}$$

where $b_{\mathbf{q}}$ is a smooth function of the type of a velocity field (Nelson, 1966) and is determined by the ground state probability density for $\phi_{\mathbf{q}}(t)$ (see Davidson, 1980). $W_{\mathbf{q}}(t)$ in equation (2) is a Wiener process satisfying

$$E(dW_{\mathbf{q}}(t)\, dW_{\mathbf{q}'}(t)) = 4\nu\delta_{\mathbf{q},-\mathbf{q}'}\, dt$$

Here E denotes the conditional expectation value with respect to the random variables $\phi_{\mathbf{q}}(t)$, and ν is the diffusion parameter. In the Davidson (1980) stochastic model, the process $\phi_{\mathbf{q}}(t)$ is a Gaussian stochastic process characterized by the expectations

$$E(\phi_{\mathbf{q}}(t))=0$$

$$E(\phi_{\mathbf{q}}(t)\phi_{\mathbf{q}'}(t'))=2\frac{\hbar}{4\omega_{\mathbf{q}}}\delta_{\mathbf{q},-\mathbf{q}'}\exp[-\alpha|t-t'|] \tag{3}$$

where

$$\omega_{\mathbf{q}}=\frac{\hbar}{2}(\mathbf{q}^2+m^2/\hbar^2)^{1/2}, \qquad \alpha=\frac{4\nu}{\hbar^2}\omega_{\mathbf{q}}$$

In the stochastic space $R_4(\hat{x})$ the field (1) may be represented as follows:

$$\phi(\hat{x})=(\hbar L^3)^{-1/2}\sum_{\mathbf{q}}\exp[i\mathbf{q}(\mathbf{x}+\mathbf{b})]\phi_{\mathbf{q}}(t+i\tau) \tag{4}$$

According to the above deduction we must average the field (4) with the distribution $\lambda(b_E^2/l^2)$. For this purpose we introduce intermediate Euclidean variables

$$b_4=\frac{2\nu}{\hbar}i\tau, \qquad x_4=it$$

and average the expression (4) with λ; thus we get

$$\langle\phi(\hat{x})\rangle=(\hbar L^3)^{-1/2}\sum_{\mathbf{q}}e^{i\mathbf{q}\mathbf{x}}\int d^4b_E\lambda(b_E^2/l^2)\exp\left(i\mathbf{q}\mathbf{b}+i\frac{\hbar}{2\nu}b_4\frac{\partial}{\partial x_4}\right)$$

$$\times\phi_{\mathbf{q}}(t)=(\hbar L^3)^{-1/2}\sum_{\mathbf{q}}e^{i\mathbf{q}\mathbf{x}}K(Q)\times\phi_{\mathbf{q}}(t) \tag{5}$$

where

$$K(Q)=\int d^4b_E\lambda(b_E^2/l^2)e^{ib_E\cdot Q}=4\pi^2\frac{l^3}{a}\int_0^\infty dy\cdot y^2\cdot\mathfrak{Z}_1(aly)\lambda(y^2)$$

$$Q=\left(\mathbf{q},\frac{\hbar}{2\nu}\frac{\partial}{\partial x_4}\right), \qquad a=(Q^2)^{1/2}=\left(\mathbf{q}^2+\frac{\hbar^2}{4\nu^2}\frac{\partial^2}{\partial x_4^2}\right)^{1/2} \tag{6}$$

Here $\mathfrak{Z}_1(z)$ is the Bessel function.

3. POSSIBLE FORMS OF DISTRIBUTIONS

Further we are interested in such a class of distributions for which $K(Q)$ are entire analytical functions of the variable Q. Now we shall calculate the function (6) for a specific form of the distributions.

Let

$$\lambda_1(y^2)=\begin{cases} c(1-y^2)^{-1/2}, & 0\leqslant y<1 \\ 0, & y\geqslant 1 \end{cases}$$

where $c=3\pi^{-2}l^{-4}/4$ is a normalization constant. Then

$$K_1(Q^2l^2)=(2\pi)^2 l^3\frac{c}{a}\int_0^1 dy\cdot y^2(1-y^2)^{-1/2}\mathcal{Z}_1(aly)$$

$$=\left(\frac{\pi}{2l}\right)^{1/2}\frac{3}{l}a^{-3/2}\mathcal{Z}_{3/2}(al)$$

Making use of the formula for $\mathcal{Z}_{3/2}(z)$,

$$\mathcal{Z}_{3/2}(z)=(2/\pi z)^{1/2}(\sin z/z-\cos z)$$

we have (see also Efimov, 1977, p. 252)

$$K_1(Q^2l^2)=3(Q^2l^2)^{-1}\left[\sin(Q^2l^2)^{1/2}/(Q^2l^2)^{1/2}-\cos(Q^2l^2)^{1/2}\right]$$

If

$$\lambda_m(y^2)$$

$$=\begin{cases} \dfrac{(\pi ml/2)^{1/2}\pi^{-2}l^{-4}(ml)^2(1-y^2)^{-1/4}}{4(\sin ml/ml-\cos ml)}\;\mathcal{Z}_{-1/2}\left(ml(1-y^2)^{1/2}\right), & 0\leqslant y<1 \\ 0, & y>1 \end{cases}$$

when

$$K_m(Q^2l^2)$$

$$=\frac{m^2l^2\left\{\sin\left[(Q^2+m^2)l^2\right]^{1/2}/\left[(Q^2+m^2)l^2\right]^{1/2}-\cos\left[(Q^2+m^2)l^2\right]^{1/2}\right\}}{\left[l^2(Q^2+m^2)\right](\sin ml/ml-\cos ml)}$$

Notice that

$$\lim_{m\to 0}\begin{Bmatrix}\lambda_m(y^2)\\ K_m(Q^2l^2)\end{Bmatrix}=\begin{Bmatrix}\lambda_1(y^2)\\ K_1(Q^2l^2)\end{Bmatrix}$$

Now let us write two more forms of the distribution:

$$\lambda_2(y^2)=\begin{cases}\pi^{-2}l^{-4}y^{-2}(1-y^2)^{-1/2}/2, & 0<y<1\\ 0, & y\geqslant 1\end{cases}$$

and

$$\lambda_3(y^2)=\alpha^2\pi^{-2}l^{-4}\exp(-\alpha y^2),\qquad \alpha>0,\ 0\leqslant y<\infty$$

The functions $K(Q^2l^2)$ corresponding to these distributions acquire the following form:

$$K_2(Q^2l^2)=\sin^2\left[\frac{l}{2}(Q^2)^{1/2}\right]\Big/\left[\frac{l}{2}(Q^2)^{1/2}\right]^2$$

and

$$K_3(Q^2l^2)=\exp(-Q^2l^2/4\alpha)$$

4. CONNECTION BETWEEN THE MARKOV EXPECTATIONS AND SCHWINGER FUNCTIONS

We now come to the main result which can be formulated as the following theorem.

Theorem. For noncoincident $\hat{x}_i[\mathbf{x}_i+\mathbf{b}_i;\ t_i+(\hbar/2\nu)(b_4)_i]$

$$\lim_{L\to\infty} E(\langle\phi(\hat{x}_1)\rangle\cdots\langle\phi(\hat{x}_N)\rangle)=S_N^M\left(\mathbf{x}_1,\frac{2\nu}{\hbar}t_1;\ldots;\mathbf{x}_N,\frac{2\nu}{\hbar}t_N\right)\quad(7)$$

where

$$S_N^M\left(\mathbf{x}_1,\frac{2\nu}{\hbar}t_1;\ldots;\mathbf{x}_N,\frac{2\nu}{\hbar}t_N\right)$$
$$=\sum_{\pi}S_2^M\left(\mathbf{x}_1,\frac{2\nu}{\hbar}t_1;\mathbf{x}_2,\frac{2\nu}{\hbar}t_2\right)\cdots S_2^M\left(\mathbf{x}_{N-1},\frac{2\nu}{\hbar}t_{N-1},\mathbf{x}_N,\frac{2\nu}{\hbar}t_N\right)\quad(8)$$

are modified N-point Schwinger functions with the times scaled by the common factor $2\nu/\hbar$. The sum over π is a sum over distinct permutations of the arguments of the S_2^M's. For the scalar field the function S_2^M has the following form:

$$S_2^M(\mathbf{x}_1, t_1; \mathbf{x}_2, t_2) = \int \frac{d^4q}{(2\pi)^4} \frac{\exp[i\mathbf{q}(\mathbf{x}_1 - \mathbf{x}_2) + iq_4(t_1 - t_2)]}{q_4^2 + \mathbf{q}^2 + m^2/\hbar^2} \times K^2\left[l^2\left(q_4^2 + \vec{\mathbf{q}}^2\right)\right] \tag{9}$$

The functions of the type (9) with the form factor $K(q_E^2 l^2)$, the physical meaning of which is discussed below, are called the nonlocal values.

Proof. In order to check this result it suffices to prove (7) for the two-point function only because the expectations of (7) will satisfy equation (8) as $\langle\phi\rangle$ is a Gaussian process.[1] So, making use of equations (4) and (5) we have

$$E(\langle\phi(\hat{x}_1)\rangle\langle\phi(\hat{x}_2)\rangle) = (\hbar L^3)^{-1} \sum_{\mathbf{q}_1, \mathbf{q}_2} \exp(i\mathbf{q}_1\mathbf{x}_1 + i\mathbf{q}_2\mathbf{x}_2) \times K(Q_1^2 l^2) K(Q_2^2 l^2) \times E(\phi_{\mathbf{q}_1}(t_1)\phi_{\mathbf{q}_2}(t_2)) \tag{10}$$

where $Q_1 = (\mathbf{q}_1, (\hbar/2\nu)(\partial/\partial x_4^1))$, $x_4^1 = it_1$ and $Q_2 = (\mathbf{q}_2, (\hbar/2\nu)(\partial/\partial x_4^2))$, $x_4^2 = it_2$. Substituting equation (3) into (10) then yields

$$E(\langle\phi(\hat{x}_1)\rangle\langle\phi(\hat{x}_2)\rangle) = L^{-3} \sum_{\mathbf{q}} \exp[i\mathbf{q}(\mathbf{x}_1 - \mathbf{x}_2)] K\left[l^2\left(\mathbf{q}^2 - \frac{\hbar^2}{4\nu^2}\frac{\partial^2}{\partial t_1^2}\right)\right] \times K\left[l^2\left(\mathbf{q}^2 - \frac{\hbar^2}{4\nu^2}\frac{\partial^2}{\partial t_2^2}\right)\right] \int \frac{dq_4}{2\pi} \frac{\exp[iq_4(t_1 - t_2)2\nu/\hbar]}{q_4^2 + \mathbf{q}^2 + m^2/\hbar^2} \tag{11}$$

Operating on the exponential in (11) by the operator $K(Q)$ and taking into account that $K(Q)$ is an entire analytical function of Q, we perform the

[1] We assume that the method of averaging (5) cannot change the physical nature of the objects and may only change the spatial structure of the object which is spread (nonlocalized) in some domain characterized by length l (see Namsrai, 1980b for detail).

limit $L \to \infty$ and obtain

$$\lim_{L\to\infty} E(\langle\phi(\hat{x}_1)\rangle\langle\phi(\hat{x}_2)\rangle) = S_2^M\left(\mathbf{x}_1, \frac{2\nu}{\hbar}t_1; \mathbf{x}_2, \frac{2\nu}{\hbar}t_2\right)$$

Thus the proof is completed. ■

Notice that what is most important in the proof is the order of differentiation with respect to the $\partial/\partial t$ in operator $K(Q)$ and of averaging with the distribution λ. It is necessary to perform first to the averaging with λ over the field (4) at every point of the space $R_4(\hat{x})$ and then (at the last step of the calculation) to act by the operator $K(Q)$. If this application of $K(Q)$ would take place in some intermediate step of the calculation, for example, if $K(Q)$ would act on the function $\exp(-\alpha|t_1 - t_2|)$, then the obtained result would correspond to the usual theory (i.e., local theory). Indeed,

$$K\left[l^2\left(\mathbf{q}^2 - \frac{\hbar^2}{4\nu^2}\frac{\partial^2}{\partial t_1^2}\right)\right]K\left[l^2\left(\mathbf{q}^2 - \frac{\hbar^2}{4\nu^2}\frac{\partial^2}{\partial t_2^2}\right)\right]\exp(-\alpha|t_1 - t_2|)$$

$$\equiv K^2(-m^2l^2/\hbar^2)\exp(-\alpha|t_1 - t_2|), \qquad \alpha = \frac{4\nu}{\hbar^2}\omega_{\mathbf{q}}$$

but the function $K(Q)$ is normalized so that $K(-m^2l^2/\hbar^2) = 1$.

Notice that for a particular value of the parameter $\nu = \hbar/2$, which is the value used by Guerra and Ruggiero (1973), we obtain the modified (or nonlocal) Schwinger function (9). In the limit $l \to 0$ this function becomes the local one and therefore we come exactly to the Guerra and Ruggiero result.

5. DERIVATION OF THE EFIMOV NONLOCAL THEORY

Owing to the work of Davidson (1980) we can suppose that in quantum field theory ν is not an observable and that measurable quantities are independent of ν. Therefore one may consider continuation of ν into the complex plane, as it was done in the paper of Davidson (1979b).

In order to show the compatibility of our theory with special relativity, we continue the expectations to the point

$$\nu = i\hbar/2$$

denoting the analytically continued expectations by E_ν. One finds easily

$$E_{\nu=i\hbar/2}(\langle\phi(\hat{x})\rangle\langle\phi(\hat{y})\rangle) = i\int\frac{d^4q}{(2\pi)^4}\frac{\exp[iq_\mu(x^\mu - y^\mu)]}{[q^2 - m^2/\hbar^2 + i\varepsilon]}V(-q^2l^2) \quad (12)$$

where $V(-q^2l^2)=[K(-q^2l^2)]^2$, $q^2=q_0^2-\mathbf{q}^2$. Comparing (12) with the usual Green's function of the nonlocal quantum field theory (Efimov, 1977) we get

$$E_{\nu=i\hbar/2}(\langle\phi(\hat{x})\rangle\langle\phi(\hat{y})\rangle)=-i\mathcal{D}_c(x-y)$$
$$=\langle 0|T(\phi_n(x)\phi_n(y))|0\rangle$$

Here

$$\mathcal{D}_c(x)=\int\frac{d^4q}{(2\pi)^4}\frac{\exp(iq_\mu x^\mu)}{m^2-q^2-i\varepsilon}V(-q^2l^2)$$

is the nonlocal causal Green's function and T denotes the Wick time ordering of the operators of nonlocal field $\phi_n(x)$ constructed by nonlocal distributions $K(l^2\Box)$ ($\Box=-\partial^2/\partial x_0^2+\partial^2/\partial\mathbf{x}^2$) (Efimov, 1968). Thus we come to the Efimov nonlocal theory. In a nonlocal theory with the form factors $V(-q^2l^2)$ belonging to the class of entire analytical functions, there exists an intermediate (or subsidiary) regularization procedure which permits changing of the contour integration in (12) to a necessary domain in the complex plane q_0 (i.e., plans I and III) (see Efimov, 1977, for details).

By analogy with the two-point Green's function for the N-point Green's functions, the following equations are valid:

$$E_{\nu=i\hbar/2}(\langle\phi(\hat{x}_1)\rangle\cdots\langle\phi(\hat{x}_N)\rangle)=\langle 0|T(\phi_n(x_1)\cdots\phi_n(x_N))|0\rangle$$
$$E_{\nu=-i\hbar/2}(\langle\phi(\hat{x}_1)\rangle\cdots\langle\phi(\hat{x}_N)\rangle)=\langle 0|T^*(\phi_n(x_1)\cdots\phi_n(x_N))|0\rangle \qquad (13)$$

where T^* is the Wick antitime ordering of operators $\phi_n(x)$. In the local case the corrections similar to (13) have been obtained by Davidson (1980).

The main restrictions in the choice of form factors $V(-q^2l^2)$ as entire analytical functions arise from the fundamental principles of theory, i.e., from unitarity (Alebastrov and Efimov, 1973) and causality (Alebastrov and Efimov, 1974).

The physical meaning of form factors consists in changing a form of the potentials between interacting fields (for example, the Coulomb and Yukawa laws) at small distances and in making the theory finite in each order of the perturbation theory in coupling constant (Efimov, 1977). The question about a possible unique choice of the form factors (in our case of distributions $\lambda(b_E^2/l^2)$ was discussed by Efimov (1977) (see also Papp, 1975).

6. CONCLUSION

The free scalar field in our model is equivalent to the nonlocal Markov field whose corrections are obtained from the nonlocal Schwinger functions in which all times are scaled by a common factor $2\nu/\hbar$. The hypothesis about the indeterminate nature of ν in quantum field theory makes it possible to obtain the Efimov nonlocal theory. Thus in our model the Euclidean Markov field and quantum field theory may be constructed in which no cutoff procedure is involved.

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